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Analysis and model of the crack bridging mechanisms in a ductile fiber reinforced ceramic matrix composite

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The force resisting the opening of a crack in a brittle matrix composite that is bridged by ductile fibers was studied (*Acta Mater.* **46**(18) (1998) 6381; *Acta Mater.* **45**(9) (1997) 3609) to gain a generic understanding of the crack-bridging process by ductile reinforcements. The matrix was alumina, initially containing a parallel array of fine cylindrical holes. Molten Al was cast into the holes to produce the fibers in situ. A crack was gently introduced to traverse the specimen. The matrix halves were pulled apart in a controlled manner to open the crack. The resisting force increased proportionally to the crack opening over a wide range until a force plateau was reached. Thereafter the force diminished very gradually until failure intervened. Analysis of this counter-intuitive behavior indicated that the excellent adhesion between the fiber and the matrix in combination with the large thermal expansion mismatch must have led to extensive but spotty debonding already from the start of the start of the crack opening. In spite of the well-known ductility of the fibers, the bridging showed quasi-elastic behavior over much of the crack opening. Necking appeared to be suppressed until the separation approached failure. Detailed modeling is offered to provide interpretation of this observed behavior.
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Nomenclature

The following quantities are defined:

D	=	separation between matrix crack faces
e_c	=	strain in composite far from crack
e_{f}	=	fiber strain far from crack
$e_{\mathrm{f}}(0)$	=	fiber strain at crack
e_{m}	=	matrix strain far from crack
E_{c}	=	Young's modulus of composite
E_{f}	=	" " "ligament
E_{m}	=	" " "matrix
K	=	Bulk modulus
L	=	nominal distance over which initial fiber decohesion occurred
n	=	strain hardening exponent
R	=	transverse dimension of ligament
v	=	volume fraction of ligaments
$s(0)$	=	nominal bridging stress on ligament
$s(z)$	=	tensile stress in ligament or matrix at location z
S	=	applied stress on composite far from crack
S_{app}	=	externally applied stress to ligament
S_0	=	strain hardening coefficient
S_{y}	=	tensile yield stress of ligament
S_θ	=	residual maximum thermal stress in ligament
u	=	total net distance between ceramic crack face and plane of incipient crack
u_{f}	=	elastic elongation of fiber at $z=0$
u_{m}	=	retraction of matrix crack face where $z \sim 0$
z	=	distance measured from plane of incipient fracture
Γ	=	work of decohesion
ν	=	Poisson ratio
τ	=	frictional shear stress

Subscripts f, m indicate ligament, matrix
The following general relationships are used:

$$\mathrm{Ec} = vE_{\mathrm{f}} + (1-v)E_{\mathrm{m}} \tag{1a}$$

$$S = vS_{\mathrm{f}} + (1-v)S_{\mathrm{m}} = vS_{\mathrm{f}}(0) + (1-v)S_{\mathrm{m}}(0) \tag{2a}$$

$$\mathrm{d}\sigma/\mathrm{d}z \bullet Area = \tau \bullet Perimeter \tag{3a}$$

Strain Hardening Law:

$$S = S_y + S_0\varepsilon^n \tag{4a}$$

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in which S_y is the yield stress, S_0 a constant, ε the plastic strain, and n the hardening exponent.

1. Introduction

Brittle ceramic matrix composites can be substantially toughened by the incorporation of ductile phases which bridge across a matrix crack and then elongate as the crack opening increases. The importance of understanding the crack-bridging response has long been recognized and the numerous relevant modeling and experimental studies have been extensively documented [1–10]. The recent availability [1] of precise data regarding the force-displacement behavior of a geometrically well-defined unidirectional Al-ligament-reinforced Al_2O_3 matrix composite offers an opportunity to examine in detail the mechanisms believed to occur in the bridging process. Those data show the crack opening displacements to be a linear function of the ligament bridging stress starting at the smallest experimentally meaningful applied stress and extending to about the ductile-brittle tensile yield stress. The range of that linearity, calculated using basic mechanical principles, indicates that some kind of decoupling of the ligament from the matrix extends over an unexpectedly long distance from the face of the matrix crack. This paper considers possible known candidate mechanisms which relate crack opening to the tensile stress applied to the composite. Credibility of a given mechanism is judged against its success or failure in predicting the observed linearity or at least the absolute value of the crack opening at some well-defined standard stress. From this procedure a model emerged that assumes partial radial detachment of the ligament from the matrix wall occurs such that random discrete regions of attachment remain between the wall and the ligament; the partial detachment is related to the geometric shrinkage that accompanies relaxation of the residual thermal tensile stress. It is proposed that this process accounts for and defines the long apparent debond length. Once the yield stress level is reached an additional mechanism becomes active and augments the first one.

2. Background

Necking is a ductile deformation process, which *a priori* might be thought to be a major factor in ligament elongation. The critical stress for the onset of plasticity in the ligament depends on the material and on the geometric constraints imposed by the confining matrix. Analytical, as well as finite element computer models [3, 4] have been made of the behavior of the ductile reinforcement constituents as the matrix crack impinges on them and continues to open.

Model experiments have provided insight into the elongation processes. Lead (Pb) was used in a pair [5, 7] of instructive studies in which the molten metal was cast into a cylindrical glass capillary or between glass plates to form filament-like or plate-like geometries. Upon carefully introducing a crack transverse to the ligaments, the force-elongation behavior, the development of the necks at the region of fracture, and debonding of the lead from site of the glass fracture could be observed. The results depended to a considerable extent on the adhesion between the lead and the glass as affected by coatings or cleaning procedures.

Direct microscopic observation [9] of the in-progress elongation process has shown that plastic elongation of the bridging ligaments occurs largely in the region between the ceramic crack faces during the crack opening process. Further insight into the bridging mechanisms has also been gained from analysis [11, 12] of gross force-displacement and fracture toughness data. Whereas determination of fracture toughness and R-curve behavior of a given composite system does not require knowledge of the resisting tensile stress that ligaments offer during crack opening, such knowledge would allow an *a priori* prediction of the fracture mechanics properties of the composite. Important further knowledge has come from basic studies of crack propagation in the model unidirectional composite system of Al fibers in an Al_2O_3 matrix [1, 2, 10]. By means of an involved mathematical deconvolution process, making use of the measured crack opening as a function of distance from the crack tip and from the measured fracture toughness, the underlying stress-elongation dependence for the individual fiber could be extracted [1]. Further recent observations [12] on that Al_2O_3 model system are summarized next.

2.1. Brief summary of experimental observations on Al/Al_2O_3 composite

Two variants of an Al_2O_3 composite were made in which parallel arrays of 13v/o 99.97% pure, aluminum wire-like ligaments were cast into a dense Al_2O_3 matrix containing cylindrical cavities having diameters of 130 or 340 μm. The fabrication details have been previously described [1, 10, 12]. A matrix crack that traversed across the fiber arrays was introduced into each of these composites. This allowed the force-displacement relation for the fibers to be measured directly because now only the fibers supported the force across the fracture plane. The total force divided by the total cross-sectional area of the ligaments allows the "nominal" mean fiber stress to be determined.

The matrix crack was introduced in a way that avoided crack opening. This was done by placing a row of diamond hardness indentations across the face of the specimen normal to the fiber direction. The microcracks emanating from the indentations provided the nucleus for the matrix crack which formed by gently dropping the specimen onto a table top. No pulling was involved. The crack was seen to emerge on the reverse side of the specimen. The fact that the crack was visible indicates that some finite crack opening had occurred and suggests that some debris or misalignment of the matrix microstructure may have prevented the crack faces from closing perfectly. Thus, some small residual tensile displacement must have occurred initially in creating the matrix crack. (This possibility may have a small effect on the theoretical considerations that follow.)

The crack was then forced open by piezo-electric actuators; the force resisting the crack opening was

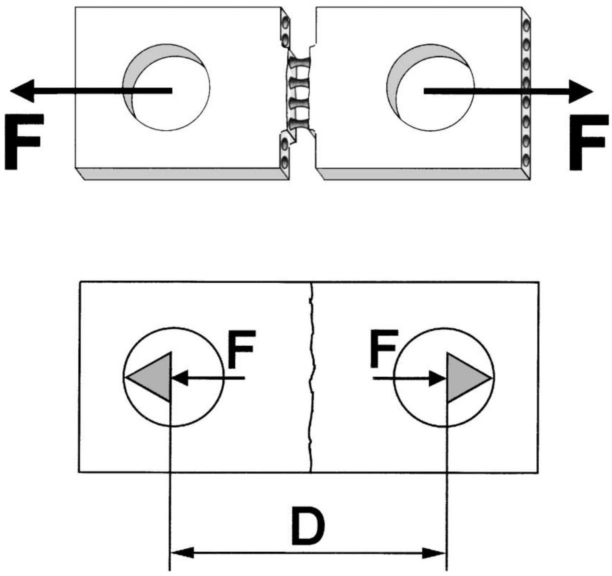

Figure 1 Schematic of system for measuring crack opening force and distance.

TABLE I Stress-crack opening data for Al_2O_3 composite

Fiber Diameter (μm)	130	340
Fiber Volume Fraction	0.13	0.13
Young's Modulus of Fiber (GPa)	72	72
Young's Modulus of Matrix (GPa)	400	400
Thermal Expansivity of Fiber (10^{-6} C^{-1})	24	24
Thermal Expansivity of Matrix (10^{-6} C^{-1})	6.6	6.6
Normal Yield Stress (MPa)	70	70
Maximum Fiber Stress (MPa)	92	90
Crack Opening (μm) at 70 MPa*	2.6	4.4
Crack Opening at Max. Stress. (μm)	5.2	11.7
Maximum Crack Opening at Fiber Failure (μm)	ca.100	ca.220
Crack Opening/Fiber Stress (μm/MPa)**	.037	.063
(Apparent Debond Length)/(Fiber Diameter)***	20	13

*70 MPa applied stress corresponds the upper bound to the linear stress vs. crack opening region and also equals the yield stress of Al.
**Average value of slope; Crack Opening = Distance between matrix crack faces = D.
***Given by (slope) $\times$ (Youngs Modulus of Fiber)/(Fiber Diameter).

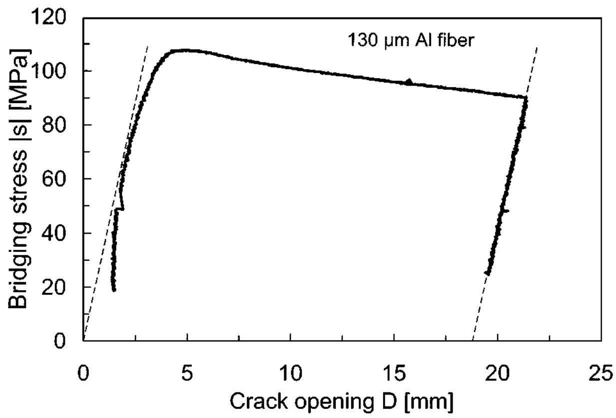

Figure 3 Ligament tensile stress (MPa) at location of crack plane versus crack gap distance (μm) for 130 μm diameter aluminum ligament showing elastic recovery upon unloading in region beyond stress maximum.

measured by a "load-cell." The experimental arrangement is shown schematically in Fig. 1 and the details have been previously given [1, 12]. The force supported by the array of crack-bridging ligaments is measurable to ± 0.1 Newtons, and the separation of the loading points can be determined to ± 0.05 μm. However, to ensure reliable seating between the loading points and the specimen, a preloading of about 10 Newtons was applied. This leads to some uncertainty in the stress levels less than 10–20 MPa.

As illustrated in Figs 2 and 3, it was found [1, 10] that (a) beginning at lowest experimentally meaningful stress level, the bridged crack in the ceramic matrix opened essentially linearly with applied stress; (b) a calculated debonding of the fiber from the matrix occurred over a distance corresponding to many ligament diameters as deduced from the measured crack openings and the known applied force, (c) the stress levels opposing further crack opening reached a maximum value when the opening reached about 3–4% of the ligament diameter; (d) increasing crack openings beyond this maximum resulted in a gradual, nearly linear, decrease of the resisting tensile stress until ligament failure occurred when the crack opening reached approximately the diameter of the ligament. The results are summarized in Table I. Ligament failure appeared to occur by a highly localized necking between the crack faces.

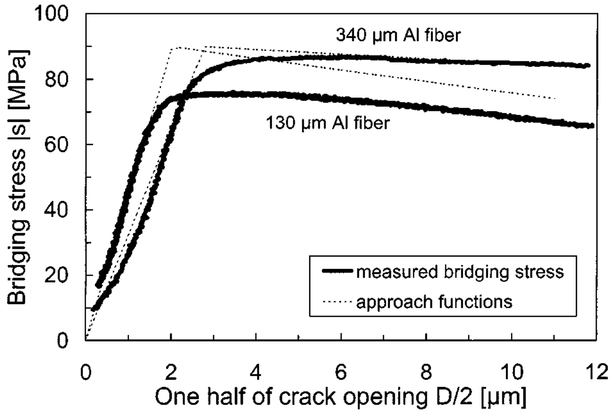

Figure 2 Ligament tensile stress (MPa) at location of crack plane versus one half of crack gap distance (μm) for 130 and 340 μm diameter aluminum ligament.

In a single experiment the composite was unloaded after having been loaded just beyond the maximum stress as shown in Fig. 3. The slope of the displacement vs. applied stress upon unloading was found to be substantially the same as that observed on the initial opening of the crack. Thus the strain appeared to be relieving elastically over the same "gage length" in unloading as in the initial loading.

No meaningful systematic deviation from stress-displacement linearity could be detected over the range from the smallest applied stress and the smallest meaningfully measurable crack opening to where the applied stress approached the yield stress of the ligament. This behavior, in part, prompted the re-examination of the details of the bridging process offered in this paper.

2.2. Approach and preliminary considerations

The introduction of a matrix crack, having even only a barely perceptible crack opening, in principle has a marked effect on the stress state in the bridging ligaments where they cross the crack opening. At the free crack (matrix) surface there can be no stress on the matrix normal to the crack plane. Without an applied stress acting on the composite, the average stress across any plane normal to the ligaments, must be zero. Thus, at the plane of the incipient crack, the axial stress in the ligaments must also be zero. This implies the creation of a stress gradient near the crack that grades from zero stress at the incipient crack to the frozen-in thermal stress value in the interior.

Simple continuum analytical models and concepts are employed that rely upon basic mechanical and physical concepts such as conservation of volume and continuity in the transmission of the applied force through the structure. The term "incipient crack" means the trajectory the crack (as yet unformed) will take once it forms. The incipient crack serves as a reference plane. The distance between two points just on opposite sides of the incipient crack is zero. Unless stated otherwise ligaments are approximated as having circular cross-sections. Terms and relationships frequently were contained in the Nomenclature section.

The unconstrained yield stress for the case of Al has been given, for example by Mataga [4] as 70 MPa. The Al ligaments in the Al_2O_3 matrix [1, 10] were produced by infiltration of molten Al into the cylindrical matrix which holds them fully constrained. This process produces a residual hydrostatic tensile stress estimated to be of the order of 1 GPa. Such a stress level is enormously greater than the unconstrained yield stress. However, as long as the filament is coherently bonded to the matrix, and no void formation occurs, no plastic deformation is theoretically possible. No experimental evidence for cavitation or other internal void formation was seen. Similar behavior was reported in other similar studies [13].

The paper examines in detail the sequence of events that occur between the passage of the matrix crack until the ligament fails. It considers in detail the initial phase of the crack opening in which the crack resisting stress increases linearly with the crack opening. A model is offered that accounts for the unexpectedly large range of the linearity. Finally, the mechanisms leading to the non-linear region at larger crack openings is discussed in detail, including (a) the stress maximum, (b) the gradual stress decrease, and (c) the failure of the ligaments between the matrix crack faces.

3. The linear regime up to the maximum in the resisting

3.1. Possible responses to initial crack opening

In as much as the linearity in the matrix crack opening vs. applied stress behavior appeared at the smallest determinable stress, various generic crack opening mechanisms are examined to see which might account for such a behavior. However, as will be shown later, none of these possible simple mechanisms appear to be consistent with the experimental observations. Thus, a new model was developed and is presented in detail later.

We consider now the geometric requirements for allowing an incipient matrix crack to open to a width D. If the crack had continued across the ligament, a space of the same width would have opened [4] in the ligament. However, ligament continuity across this gap requires that the volume needed to fill the gap be filled in with ligament material by either elastic deformation or by material flow. At the same time conservation of the ligament material must be satisfied.

Four scenarios can be envisioned for accommodating the matrix crack opening:

(1) The ligament remains firmly attached everywhere to the confining matrix wall. This requires material flow within the ligament immediately ahead of where the matrix crack impinges on the ligament but requires that the ligament remain attached to the wall immediately adjacent to the incipient crack. The flow implies that the local stresses must have risen above the yield stress allowing voids to form in the proximity of where the matrix crack impinges on the ligament.

(2) The incipient crack remains closed across the ligament, but matrix crack opening can occur some what remote from the ligament through elastic shear displacement of the matrix.

(3) Physical contact is maintained with the wall during sliding. This is typically envisioned as the case when frictional sliding occurs. A general problem with mechanisms (2) and (3) is that they predict a decrease in the level of the applied axial tensile stress with increasing distance from the crack. Such a dependence is counter to the requirement that the matrix stress must ultimately increase to the level of the thermal stress far from the matrix crack.

(4) Physical separation from the wall, i.e., debonding occurs. In this case there can be no shear coupling so that the tensile stress in the ligament must remain constant. However, this situation cannot persist over the length of the ligament or complete pull out would result.

These candidate mechanisms are tested for (a) their ability to display linearity between the nominal ligament stress and crack opening, and (b) their prediction of the crack opening D at applied stresses below the critical yield stress. Experimental determination of the slope of nominal stress vs. D dependence at stress approaching zero becomes difficult at near-zero stresses. However, the crack opening at higher stress levels can be measured with reasonable confidence. For the case of the Al/Al_2O_3 system, the largest crack opening at which linear behavior substantially still occurred was at the yield stress level S_y of the ligament. Hence, this level is selected as the standard value for comparing the calculated crack openings against the observed crack opening for the various candidate mechanisms. At that stress level the measurements of the crack opening show relatively little scatter. Thus, a comparison of this value of D/R, relative to that calculated for the candidate mechanisms, can serve as another criterion for judging the mechanisms. Comparison of their relative performances is offered after the four candidate possibilities have been discussed.

3.1.1. No debonding and no slippage

Previous studies, [3–5] including the toughening of ceramic matrix composites by the addition of sphere-like isolated ductile inclusions, considered that yielding commenced where the crack impinged on the inclusion. This process allowed the ligament to bridge across the crack and to resist further crack opening. Volume conservation required material to be redistributed to fill the gap. Calculations [3] for the early stages crack opening D indicated it to increase approximately proportional to the square of the applied stress for values up to 6 times the critical yield stress. However, crack bridging by relatively large diameter ductile ligaments is sufficiently different with respect to geometry and size to warrant reexamination of the bridging process especially during the early stages of crack opening.

Shear displacement and debonding at the matrix wall are precluded. At the earliest stages of crack opening, void formations at the intersection of the incipient crack with the ligament are needed to relieve the unrealistic large strains that would otherwise be produced. The voids also compensate for the volume of material displaced out of the region between the crack faces much in the manner previously detailed [3, 4], as is shown in Fig. 4. The previous analyses considered that material could flow plastically to fill the region between the faces. However, this mechanism appears unlikely at low applied stresses where the voids are vanishing small. Instead we consider that longitudinal elastic stretching of ligament in the region between the voids is needed to bridge between the crack faces.

In order to model the expected dependence of the crack opening D on the applied stress S we consider the case of a cylindrical ligament of radius R and a Young's modulus E in which the matrix crack opening is D. As seen from the figure, the voids form a torus having a major radius $R - r$ and a minor radius as shown in Fig. 5. The volume of material remaining between the crack faces is given by a radius $R - 2r$ and a thickness $2r$. This volume is just that provided by the material formerly contained in the voids. Because the crack faces move apart a distance D, it follows that the distance corresponding to the void diameter $2r$ before crack face opening must increase to $2r + D$ after

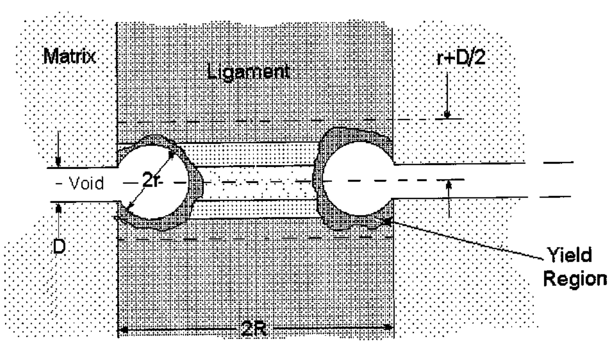

Figure 4

the opening. This corresponds to a longitudinal strain of $D/(2r + D)$ from which the stress needed to produce the crack opening D can be estimated. The final result is

$$D = (S/E)^2 R/2\pi. \tag{1}$$

When the value of $S = S_y$, $D = 10^{-6} R/2\pi$. Note that the crack opening is a quadratic function of S as was previously found for the spherical ductile inclusions.

3.1.2. Elastic shear deformation but no debonding from the wall

This mechanism is usually applied to such composites as glass-bonded inorganic fiber systems in which the elastic fiber is much stiffer than the matrix to which it is bonded. The well-known, simple shear-lag model [14–16] can be used to estimate the behavior of such a system. Neither fiber failure, debonding, or cavitation (void formation) are considered. The matrix crack remains pinned shut where it impinges on the fiber, but the crack faces separate increasingly with increasing distance from the ligament bridge. The model indicates that the external stress applied to the ligament decays exponentially with distance z from the incipient crack. Assuming the applicability of the shear lag model to the present case in which the fiber is elastically relatively soft and the matrix relatively stiff leads to the following expression for the decay constant k which has been given [16] as

$$k = \sqrt{\frac{4E_m}{E_f \cdot (1 + \nu_m) \cdot \ln(\pi/v_f)}} \tag{2}$$

where E_f and E_m are the Young's moduli for the fiber and the matrix, respectively and v_f is the volume fraction of the fibers. The dependence of the crack opening on the applied stress for the case of a square array of ligaments becomes:

$$D = 2(s(0)/kE_f)R \tag{2a}$$

in which $s(0)$ is the tensile stress in the fiber at $z = 0$. Note that in this case the opening is a linear function of the stress. The analysis for the present situation of a soft reinforcement fiber in a stiff matrix is much more complicated [17] and has not been completely developed suitable for use in the present case.

3.1.3. Shear yielding at the wall while maintaining wall contact

The appearance of a matrix crack initially induces an abrupt change in the axial stress in the ligament from the frozen-in tensile thermal stress level to zero. The shear stress at the wall can be expected to reach the threshold for yielding in the vicinity of the crack, as discussed for mechanism (1). The stress levels and the axial range over which yielding at the matrix wall can occur are complex considerations beyond the scope of this paper. However, it is sufficient that such yielding at

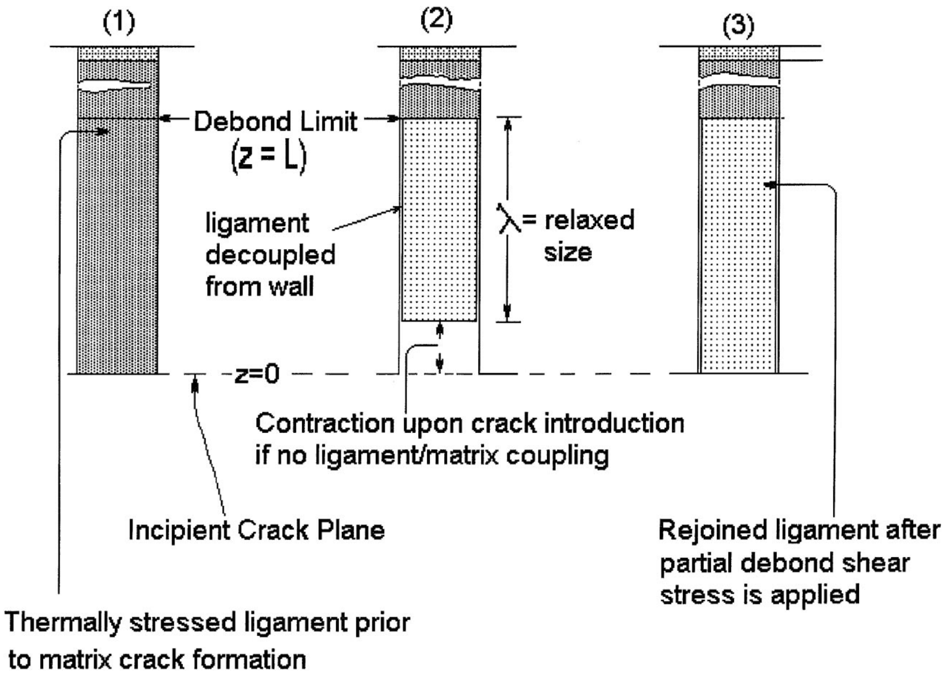

Figure 5

the wall allows slippage at the wall resisted by a shear stress S_{shear} that resists the slippage and which can be estimated to have an near-constant value equal to S_y the shear yield stress. The axial ligament stress $s(z)$ as a function of distance z from the incipient crack plane, and the ligament radius R is then given by [15]:

$$\pi R^2 \mathrm{d}s/\mathrm{d}z = 2\pi R S_y \quad \text{or} \quad s(z) = s(0) - (2S_y/R)z. \tag{3}$$

in analogy to fiber pull-out [16] with frictional coupling between ligament and the wall, $s(0)$ being the axial ligament stress at $z = 0$, and the other parameters have been previously defined and are found in the Nomenclature.

The ligament elongation u is given by the integral in which the axial strain as a function of z is $\varepsilon(z) = s(z)/\text{Ef}$:

$$u = \int_0^{z*} \varepsilon(z)\,\mathrm{d}z \tag{4}$$

If z^* is the distance at which $s(z) = 0$, then D is given by:

$$D = [s(0)^2/(2S_y E_f)]R, \tag{5}$$

in which D is a quadratic function of the applied ligament stress $s(0)$. When the applied stress $s(0)$ is raised to the level S_y, the calculated crack opening $D = (S_y/2\text{Ef})R$.

3.1.4. Ligament debonding from the matrix

Complete fiber-matrix debonding could, in principle, commence as soon as the matrix crack is formed when the relaxation of the frozen-in thermal tensile stresses allows a volumetric contraction of the ligament. Subsequent application of a tensile stress to the ligament would result in a further radial contraction due to Poisson effects. Once complete wall contact is lost over some distance z from the crack, there is no way that tensile stress can be transferred between the ligament and the matrix. Hence, the tensile stress would remain zero and ligament pullout would result [2] unless the debonding were somehow restricted to a distance L from the crack site. In that case, the stress over that distance would remain fixed at the level of the applied stress, or $s(z) =$ constant for $z < 0 < L$ and

$$e(z) = \text{constant} = s(0)/E_f = D/L \tag{6}$$

However, realistically there would probably be some frictional sliding resistance caused perhaps by wall roughness leading to interference between the ligament and the wall. However, once frictional drag commences, further incremental crack opening $\delta\ D$ would follow a quadratic response similar to that given by (5) in which S_y would be replaced by τ(friction).

3.2. Considerations specific to the Al/Al_2O_3 system

In determining the predictions for the relative crack openings D/R at an applied stress S_y, the volume fraction of ligaments is set at 0.13, the Poisson ratio at 0.3 and the ratio S_y/E of the standard applied stress at the stress level S_y to the Young's modulus E is set at 0.001. The characteristics for the various mechanisms given in Table II and are discussed below:

Mechanism 3.1.1 can be ruled out on the basis of the discrepancies with respect to both the functionalities and the D/R ratios.

In mechanism 3.1.2 the ligament is elastically shear bonded to the matrix and the functionality agrees with that observed experimentally. Introducing the physical property values and setting

TABLE II Observed vs. the values predicted by the candidate mechanisms for Al/Al_2O_3

Mechanism	Relevant Equation	Functionality	D/R at Al Yield Stress (70 MPa)
EXPERIMENT		Linear	0.04 ± 0.01
(3.1.1.)Ideal Adherence	1.	Quadratic	$3\ 10^{-7}$
(3.1.2.)Elastic Shear	2a.	Linear	0.0006
(3.1.3)Plastic Shear	4.	Quadratic	0.001
(3.1.4.)Debond, then Friction	4, 6.	Linear then Quadratic	***

Vf = 0.13 gives $k = 3.1$ Thus, D/R at $S = S_y$ is calculated to be $(2/3.1)(s/\mathrm{Ef}) = .0006$ compared with the observed value 0.04. This 60-fold discrepancy cannot be attributed to the uncertainties in the shear lag model. Hence this discrepancy indicates that this was not the operable mechanism.

The mechanism 3.1.3 that proposes shear yielding at the wall fails both with respect to the functionality and the D/R value although in this case the discrepancy is somewhat less than in the above case..

Mechanism 3.1.4 is ambiguous. The crack opening would be a linear function of the applied stress but only if there were a finite region over which total debonding occurred. Once frictional drag commenced, the response would no longer be linear. Thus, a linear response followed by a quadratic response would be expected.

3.3. Determination of the debond length

The crack opening $D = 2u$ was observed to be proportional to the nominal applied stress $s(0)$ at $z = 0$. Equation 4 is fundamental to calculating the elongation needed to fill the gap between the crack faces. This equation suggests two extreme possibilities for the observed behavior: (1) The integrand $\varepsilon(z)$ remains constant, but the upper bound of the integral increases linearly with increasing applied stress S, or (2) The upper bound of the integral is fixed, but the integrand $\varepsilon(z)$ increases linearly with increasing applied stress. Possibility (1) can be dismissed because it leads to a contradiction, viz., it allows the applied stress to be increased while at the same time requiring that $\varepsilon(z)$ remain fixed. Hence, only possibility (2) is viable.

The debond length L serves as a gage length between which the ligament can elongate in accordance with Equation 6. The value of L was found to be independent of $s(0)$, but dependent on the ligament radius. For instance when the applied stress on the fiber had the intermediate value 60 MPa, the measured crack openings were 2.2 and 3.8 μm, respectively for the 130 and the 340 μm fibers, and the calculated values for L are 2.6 and 4.4 mm, i.e., 20 and 13 times the fiber diameters, respectively! That is, the apparent gage length is much greater than the ligament diameter. This implies that as soon as the matrix crack appears, substantial non-reversible debonding must occur. At distances from the crack larger than L, the ligament returns to being in intimate contact with the matrix. Total indefinite debonding is not possible [2] because in that case the ligament would simply be pulled out of its "socket." It is not unreasonable that debonding can occur over a long distance considering that the initial thermal stress and stored elastic energy are so large. Speculation of the kind of coupling that might be limited to a particular distance from the crack plane is offered below.

3.4. Residual thermal stresses and ligament shrinkage

When the crack in the matrix first forms, the axial stress in the matrix vanishes. So the stress in the ligament at the crack plane must also vanish in the absence of an applied stress. Assuming a temperature difference of about 600°C between room temperature and the onset of freezing of the aluminum and assuming the metal to be strongly bonded to the alumina matrix leads to an estimated residual "frozen in" thermal strain of about 1% or a hydrostatic tensile stress of about 1 GPa. Release from such a state of high tensile stress and strain would result in shrinkage. This is expected to produce a gap or void formation at the periphery, shown schematically in Fig. 5. As already discussed, there must nevertheless be some coupling between the ligament and the wall. For example, local regions might still adhere to the wall. Experimental evidence for such vestiges of adherence is seen near the sites of ultimate necking failure as shown in Fig. 6a and b. These adherences could provide shear coupling between ligament and the matrix. How such a partial debond could arise is discussed next.

4. Proposed mechanism for ligament stress redistribution and crack opening behavior

The degree of radial shrinkage will depend on the local ligament tensile stress. The debonding process terminates when not enough energy can be released to "pay" for the debonding process and the creation of the ductile adherents shown in Fig. 6. The internal ligament stress $s(z)$ ranges from zero stress at the crack face to the frozen-in thermal stress S_θ far from the crack. Although without further knowledge of the details of the formation of the adherences it is not possible to provide *a priori* the dependence $s(z)$ of the stress along the fiber, a plausible approximation can be offered by simply devising an empirical equation such as:

$$s(z) = S_\theta \tan h(bz) + A(z - az^2)\exp(-Bz), \quad (7)$$

which has the properties $s(z = 0) = 0$ and $s(z = \mathrm{inf.}) = S_\theta$; A, B, and b are constants to be determined. However, it is sufficient that such a stress dependence could exist; the exact form and details of $s(z)$ are not needed for the discussion that follows in the next section.

This empirical equation provides a rationale for the large deduced values of the L/R ratio obtained from analysis of the experimental data. An apparent shear stress τ can be calculated from Equation 7 through $(R/2)\ \mathrm{d}s(z)/\mathrm{d}z$. The various adjustable constants can

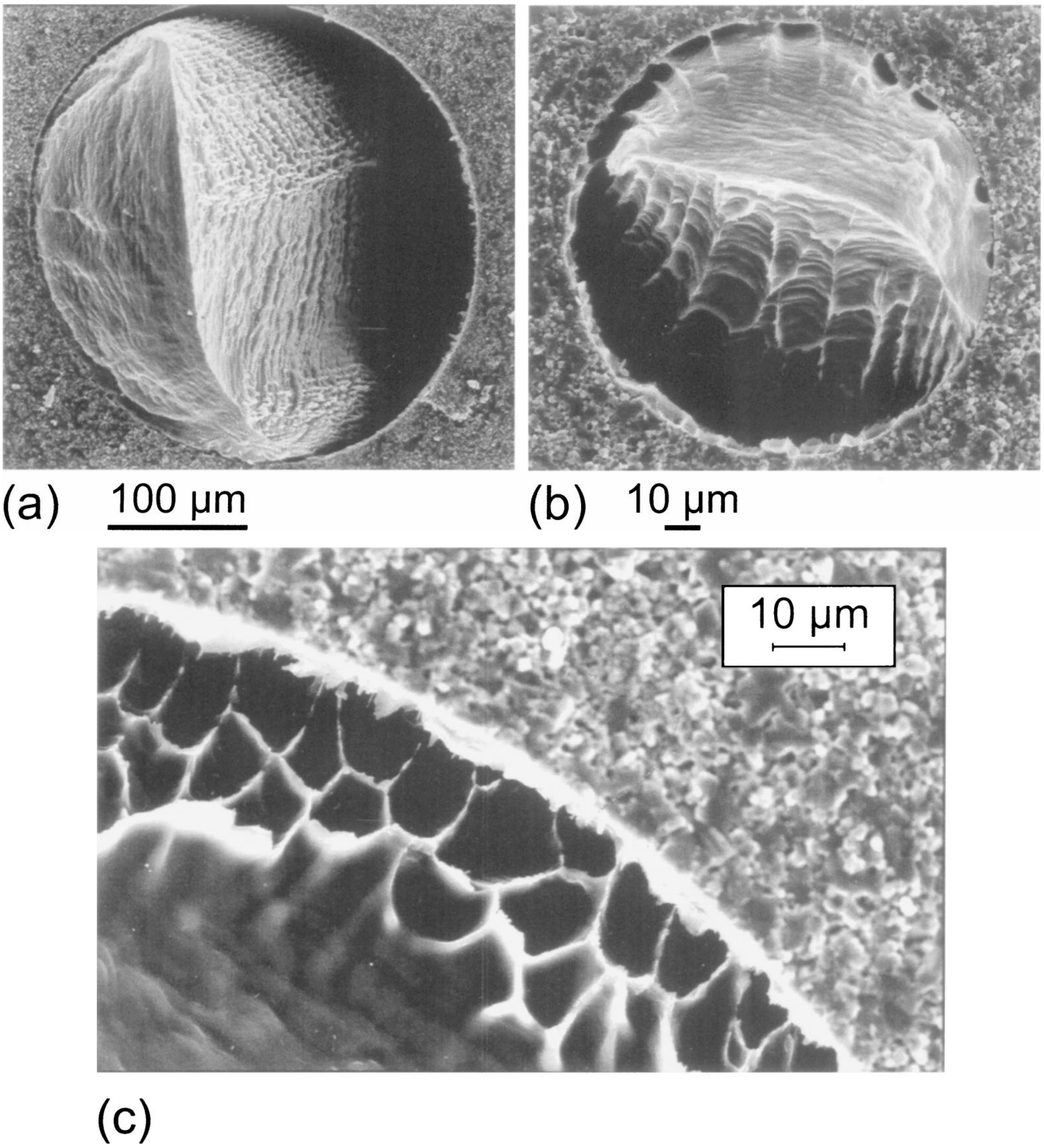

Figure 6

be fixed by requiring that (1) the computed elongation shows the fiber remains intact as it bridges the incipient crack, (2) the ligament stress and rate of stress change with distance so as to join smoothly with the stress at the boundary where the "partial debond" region comes to an end, and (3) the rate of stress increase at the incipient crack agrees with the shear stress estimated on the basis of the partial debond mechanism. Applying this procedure to Equation 7 showed that real solutions were obtainable yielding values of L/R as great as 7. Other empirical equations can probably be devised that give larger L/R ratios. Physically the adherent formation and elongation process can be expected to evolve so as to maximize the extent of decoupling within limits, such as those cited above. Questions that arise in considering the plausibility of this mechanism are considered next, viz.: (1) where does the energy come from to pay for this mode of debonding; (2) how is such debonding affected by the application of applied stress; and (3) how does this affect the dependence of crack opening on applied stress.

4.1. Energy needed for decoherence

Prior to matrix cracking the energy per unit volume stored in the ligament was $(S_\theta)^2/2K$. As the ligament stress is reduced to $s(z)$, the stored energy is reduced to $s(z)^2/2K$, where K is the bulk modulus. This difference in energy is needed to pay for Γ, the work of decohesion per unit length. Noting that $(S_\theta)^2 - s(z)^2 = (S_\theta + s(z))$ $(S_\theta - s(z))$ and assuming $s(z)_\theta \simeq S$, one can write approximately

$$\left(\pi R^2 \bullet 2S_\theta \Sigma\right)/2K <= 2\pi R\Gamma \qquad (8)$$

in which $\Sigma = S_\theta - s(z)$ leading to the particular value ${}_0\Sigma = 2K\Gamma/R_0 S$ which define the upper bound to $s(z)$

that allows decohesion. The value of z where $s(z)$ satisfies Σ_0 defines the decohesion limit L.

4.2. Effect of the applied stress on the decohesion length

Applying a stress Sapp to this initially externally unstressed system merely imposes Sapp onto the initial self-stressed state. The energy required for decohesion, given by the right hand term in Equation 8, remains unchanged. The stored energy per unit length prior to extension of the decohesion region is $(S_\theta + \text{Sapp})^2$ and after extension the corresponding energy will be reduced to $(s(z) + \text{Sapp})^2$ where $s(z)$ is the ligament stress at z prior to the adding the applies stress. Applying Sapp has little effect on the energy difference. Thus, the decohesion length L remains essentially unaffected by Sapp.

4.3. Dependence of applied stress on crack opening

We now consider the crack opening response of the system as a function of Sapp. We measure the elongation $u(\text{Sapp})$ relative to the initial length when $\text{Sapp} = 0$. In accordance with Equation 3, this is given formally by:

$$u(\text{Sapp}) = \int_0^L \frac{(\text{Sapp} + s(z))}{K} \cdot \text{dz} - \int_0^L \frac{s(z)}{K} \cdot \text{dz} = \int_0^L \frac{\text{Sapp}}{K} \cdot \text{dz} \qquad (9)$$

It may be recalled that K is the bulk modulus of the fiber. Thus, even though L and $s(z)$ are not specifically defined, and provided that these quantities are not affected by Sapp, the system acts as if it were unbonded over a distance L from the crack. The functional dependence of the internal stress $s(z)$ in the self-stressed state that results from the formation of the matrix crack is unimportant because $s(z)$ cancels out.

5. Neck formation and non-linear crack extension

In the radially unconfined region between the matrix crack faces, there is no geometric constraint that affects the yield stress; so yielding can occur when the conventional threshold yield stress is reached. In the yielded region the strain state becomes hydrostatic, i.e. the radial strain and the axial strain become equal. This has the effect of increasing the radial shrinkage and increasing axial extension. The situation becomes similar to pullout of a ductile fiber from a brittle matrix [2] in which complete debonding becomes possible in principle. Note that in the present experiments, the crack opening but not the stress were imposed. Because the lowest stress at which yielding can initiate is where there is no geometric constraint, i.e., in the crack bridging region, neck formation begins there. The overall measured elongation of the ligament is equal to the elongation of the neck plus the elastic elongation of the ligament remaining in the matrix.

The neck is geometrically constrained by the requirement of conservation of volume of the ligament. We follow Mataga [4] in approximating the neck geometry to be a parabola of revolution. With a minor recasting of the geometric variables, the radius $r(z)$ is given by

$$r(z) = R - a + B(z/R)^2,$$

in which a is the distance that the "waist" has been pinched in at $z = 0$, and B is a shape parameter to be determined. The derived longitudinal radius of curvature of the neck is $2B$ at $z = 0$, and the value of z when $r(z) = R$ is $Z = R\ \text{Sqrt}(a/B)$. Invoking conservation of volume of the ligament as it undergoes necking, the following approximate relationship is found that is valid for $a \ll R$,

$$a/R \sim (3/8)(D^*/Z).$$

That amount of the crack opening attributable only to necking is designated D^*.

Although the necking process reduces the cross-sectional area, tending to reduce the axial force transmitted by the ligament, the strain hardening more than compensates for this effect up to a maximum at which ε (max) $\sim n/(2+n)$, where n is the strain hardening exponent. By making use of the Bridgman result [18], the strain hardening relationship for plastic deformation, and noting that ε (plastic, radial) $\sim a/R$, the contribution D^* relative to the total crack opening D is given through

$$(3D^*/4D)^n = (\sigma(0) - S_y)/S_y,$$

in which n is the strain hardening exponent, S_y is the usual yield stress and $\sigma(0)$ is the nominal applied tensile stress. This relationship is valid over the range in which the force transmitted through the neck continues to increase with increasing D. Finally when neck decreases to the point where a $\sim\varepsilon$ (max)$R \sim n/(2+n)R$, further plastic elongation results in a decrease of the force transmitted to the neck, i.e., increasing D causes

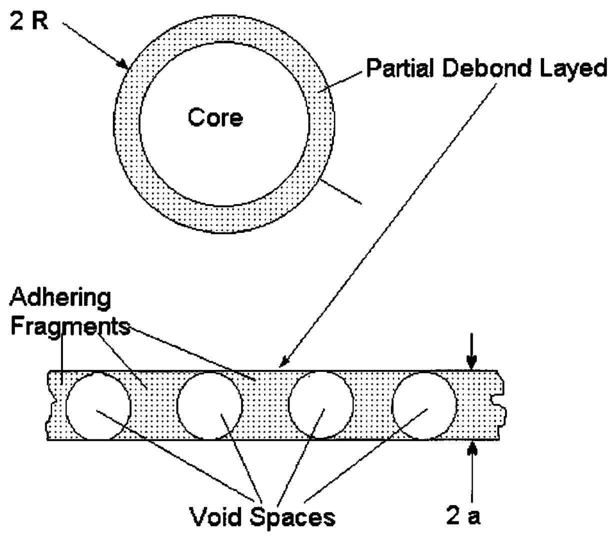

Figure 7

a decrease in the apparent applied stress. Note that this plastic deformation is limited to the immediate region of the crack opening, because the rest of the ligament is still retained within the confines of the matrix. This portion of the ligament is under an axial elastic tension that is well below the yield stress. Hence, unloading the sample at this point will simply result in an elastic contraction of both the regions which had undergone yielding and those that remained elastic throughout the process. Continuing to increase D will elongate the neck, causing the waist to decrease until failure of the ligament finally occurs either in a "chisel point" or by reaching the limit to strain hardening and rupturing.

6. Summary of the overall process

The overall crack opening process can be viewed as occurring in 6 stages as shown schematically in Fig. 8. Stage (I) corresponds to the situation in the ligament prior to the arrival of the matrix crack, which is shown as the incipient crack. In Stage (II) the matrix has cracked, decohesion has occurred up to the debond limit and the partial debonding is suggested by the islands of attachment. No external stress has been applied, so that the crack faces do not separate. In Stage (III) a modest tensile stress that is substantially less than the yield stress for the ligament is applied causing the crack faces move to apart. Stage (IV) represents the situation when the applied stress is just less than the yield stress, whereas Stage (V) corresponds to when applied stress is increased to just in excess of the yield stress. Finally, in Stage (VI) the extension is increased sufficiently to cause sufficient necking so that further crack opening will result in a decrease of tensile *force* that the neck can support. Further opening will result in further decrease of the neck diameter and ultimately to ligament failure.

7. Discussion and conclusion

The usual composite models were found to be inadequate for accounting for the crack opening and stress behaviors of the unidirectional Al/Al_2O_3 composite system. Accordingly a new hybrid model of ligament/matrix coupling is proposed in which fibril-like attachments in the region connect the matrix to the ligament core. This proposed structure is assumed to arise from the combination of a very large residual thermal stress, its abrupt relaxation in the vicinity of the matrix crack, the strong adhesion between the ligament and the matrix, and the ductility of the ligament. This structure accounts for the quasi-elastic linear behavior when the applied stress is less than the yield stress of the ligament. It is also accounts for the behavior in the later stages of crack opening, and for the appearance at failure.

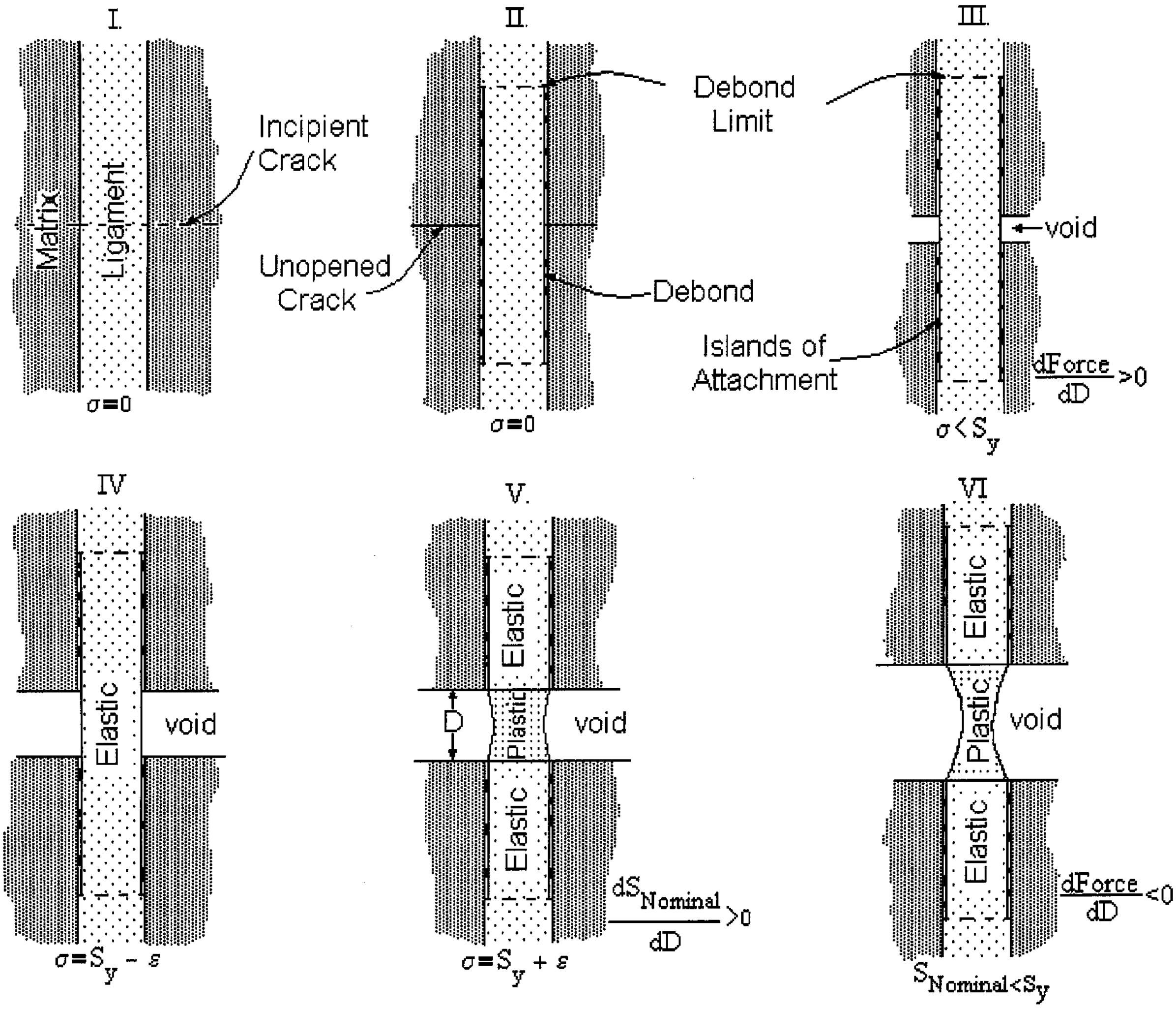

Figure 8

Other investigators have suggested that plastic tearing [19] or that sub-grain rotation [20] of the ligament material occurs at the wall. Experimentation is needed to determine the actual details of the interface decohesion process. Definitive direct micrography of the debond region ligament/matrix wall has not yet been achieved. The debonded region was deduced to extend far into the matrix. It is possible that the unexpectedly extensive debonding is attributable to the atypically large ligament diameters necessitated by the melt-cast process for the specimen fabrication. Nevertheless the observed phenomena are believed to be generally valid and should provide guidance in estimating the behavior of as-yet un-studied candidate systems involving ductile metal reinforcements of brittle matrix composites.

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